Beyond KKT

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The KKT conditions are the best known optimality conditions for constrained optimization. They correspond to the classical Lagrangian conditions for equality constrained problems. However, local minimizers (or even global minimizers) do not always satisfy KKT. For example, the obvious minimizer of "Minimize x subject to $x^2 = 0$" does not satisfy KKT. In fact, KKT conditions are necessarily satisfied by local minimizers only when some "constraint qualification" holds. As a consequence, one could guess that algorithms for constrained optimization test, at each iteration, whether KKT or "the negation of a constraint qualification" takes place. In both cases we would have a good candidate for local minimizer. However, no practical known algorithm tests the non-fulfillment of constraint qualifications. Practical algorithms only test the fulfillment of KKT. In this talk, I will explain why this is the case. The reason is that, although local minimizers do not necessarily satisfy KKT, every local minimizer satisfies "Approximate KKT conditions" that are very adequate for numerical testing in the context of iterative methods. The convergence analysis of iterative optimization methods towards non-KKT points is a new field in numerical optimization theory to which very few research has been dedicated.