Look-Back GMRES(m) for
Solving Large Nonsysmmetric Linear Systems

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In recent years, the Krylov subspace methods have been extensively researched for large and sparse linear systems of the form

$$Ax = b, \quad A \in \Re^{n \times n}, \quad x, b \in \Re^{n}, \tag{1}$$

where the coefficient matrix A is assumed to be nonsymmetric and nonsingular.

The GMRES method [2] is one of the most popular Krylov subspace methods for solving nonsymmetric linear systems. While the GMRES method shows a good convergence, it has some difficulties in storage and computational costs. To remedy these difficulties, restarted version of the GMRES method, the GMRES(m) method [2], is widely used as a practical choice.

The technique the so-called ***restart*** can be described as follows: First, we choose the restart frequency m and the initial guess $\boldsymbol{x}_0^{(1)}$ of the 1st restart cycle, and then, the 1st restart cycle is started. In the l-th restart cycle, the linear system (1) is solved (approximately) by m iterations of the GMRES method with the initial guess $\boldsymbol{x}_0^{(l)}$, and get the approximate solution $\boldsymbol{x}_m^{(l)}$. After that, the initial guess of the (l+1) th restart cycle is updated, i.e., $\boldsymbol{x}_0^{(l+1)} := \boldsymbol{x}_m^{(l)}$, and go to the ($l$+1)th restart cycle. This restart cycle is repeated until convergence.

From the investigation on how to update the initial guess at each restart cycle for efficient convergence, we have recently proposed a GMRES(m) method with ***unfixed update*** instead of the fixed update $\boldsymbol{x}_0^{(l+1)} := \boldsymbol{x}_m^{(l)}$ [1].

In this talk, we will analyze the unfixed update based on the residual polynomials, and then a GMRES(m) method with ***Look-Back-type restart*** will be proposed in the framework of unfixed update. The numerical experiments indicate that the GMRES(m) method Look-Back-type restart is more efficient than the GMRES(m) method in many cases.

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[2] Y. Saad and M. H. Schultz, GMRES: A Generalized Minimal Residual Algorithm for Solving Nonsymmetric Linear Systems, SIAM J. Sci. Stat. Comput., 7(1986), pp. 856-869.