Recursively Deflated PCG for mechanical problems

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Abstract

We consider the simulation of displacements under loading for mechanical problems. Large discontinuities in material properties, such as encountered in composite materials, lead to ill-conditioned systems of linear equations. These discontinuities give rise to small eigenvalues that may negatively affect the convergence of the Preconditioned Conjugate Gradient (PCG) method.

This paper considers the Recursively Deflated Preconditioned Conjugate Gradient (RDPCG) method for solving such systems. Our deflation technique uses as deflation space the rigid body modes of sets of elements with homogeneous material properties. We show that in the deflated spectrum the small eigenvalues are mapped to zero and no longer negatively affect the convergence. We justify our approach through mathematical analysis and we show with numerical experiments on both academic and realistic test problems that the convergence of our RDPCG method is independent of discontinuities in the material properties. In many known examples there are only two materials involved: a weak and a stiff material. Our Recursively Deflated PCG method can be applied to a whole cascade of weaker and stiffer materials.

We consider asphalt concrete as an example of a composite material. It consists of a mixture of bitumen, aggregates and air voids. Obviously the difference between the stiffness of bitumen and the aggregates is significant, especially at high temperatures.

We simulate the response of a composite material that is subjected to external forces by means of small load steps. By using the FE method we obtain the corresponding stiffness matrix. Solving linear system $Ku = f$ is the most time consuming part of the FE simulation. The stiffness matrix K is symmetric positive definite. We have shown in [1] that the number of iterations to convergence for PCG is highly dependent on the number of aggregates in a mixture as well as the ratio of the E moduli. Increasing the number of aggregates introduces correspondingly more small eigenvalues in stiffness matrix K. The jumps in the E moduli are related to the size of the small eigenvalues. Recently this approach has been compared and combined with Algebraic Multigrid (AMG) methods [2]. It appears that the combination leads to faster methods than the the standalone AMG solver.

References

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[2] T.B. Jönsthövel, M.B. van Gijzen, C.Vuik, and A. Scarpas. On the use of rigid body modes in the deflated preconditioned conjugate gradient method. *SIAM Journal on Scientific Computing*, 2011. to appear.