IDR AS A DEFLATION METHOD

GERARD L.G. SLEIJPEN*

Abstract. IDR (Induced Dimension Reduction) is a family of efficient iterative methods for the numerical solution of large non-symmetric systems $\mathbf{Ax} = \mathbf{b}$ of linear equations. Examples of IDR methods are Bi-CGSTAB [8], BiCGstab(ℓ) [3] and BiCGStab2 [2] and the more recent methods, IDR(s) [10], [6] and IDRstab [4], [7]; Bi-CGSTAB is equivalent to the original IDR method [9]. These IDR methods rely on short recurrences and all iteration steps are equally fast. Often the convergence (in terms of the number of multiplications of vectors by the matrix $\mathbf{A}$) of the recent versions is comparable to the convergence of GMRES [5].

In this talk the IDR method is interpreted in the context of deflation methods. It is shown that IDR can be seen as a Richardson iteration preconditioned by a variable deflation-type preconditioner.

The main result of this talk is the IDR projection theorem, which relates the spectrum of the deflated system in each IDR cycle to all previous cycles. The theorem shows that this so-called active spectrum becomes increasingly more clustered. This clustering property may serve as an intuitive explanation for the excellent convergence properties of IDR. These remarkable spectral properties exist whilst using a deflation subspace matrix of fixed rank.

This is joint work [1] *with Tijmen Collignon and Martin van Gijzen*†.

REFERENCES

[1] Tijmen Collignon, Gerard L. G. Sleijpen, and Martin B. van Gijzen, *Interpreting IDR(s) as a deflation method*, Report ISSN 1389-6520, Department of Applied Mathematical Analysis, Delft University of Technology, the Netherlands, October 2010.

[2] Martin H. Gutknecht, *Variants of BICGSTAB for matrices with complex spectrum*, SIAM J. Sci. Comput. **14** (1993), no. 5, 1020–1033. MR 1232173

[3] Gerard L. G. Sleijpen and Diederik R. Fokkema, *BiCGstab(ℓ) for linear equations involving unsymmetric matrices with complex spectrum*, Electron. Trans. Numer. Anal. **1** (1993), 11–32 (electronic only). MR 94g:65038

[4] Gerard L. G. Sleijpen and Martin B. van Gijzen, *Exploiting BiCGstab(ℓ) strategies to induce dimension reduction*, SIAM J. Sci. Comput. **32** (2010), no. 5, 2687–2709.

[5] Peter Sonneveld, *On the convergence behaviour of IDR(s)*, Preprint 10-08, Technology, Delft, The Netherlands, March 2010.

[6] Peter Sonneveld and Martin van Gijzen, *IDR(s): a family of simple and fast algorithms for solving large nonsymmetric systems of linear equations*, SIAM J. Sci. Comput. **31** (2008), no. 2, 1035–1062.

[7] Masaaki Tanio and Masaaki Sugihara, *GBi-CGSTAB(s,l): IDR(s) with higher-order stabilization polynomials*, J. Comput. Appl. Math. **235** (2010), no. 3, 765 – 784.

[8] Henk A. van der Vorst, *Bi-CGSTAB: A fast and smoothly converging variant of Bi-CG for the solution of nonsymmetric linear systems*, SIAM J. Sci. Statist. Comput. **13** (1992), no. 2, 631–644. MR 92j:65048

[9] Piet Wesseling and Peter Sonneveld, *Numerical experiments with a multiple grid- and a preconditioned Lanczos type method*, Lecture Notes in Mathematics, vol. 771, pp. 543–562, Springer Verlag, Berlin, Heidelberg, New York, 1980.

[10] Man-Chung Yeung and Tony F. Chan, *ML(k)BiCGSTAB: A BiCGSTAB variant based on multiple Lanczos starting vectors*, SIAM Journal on Scientific Computing **21** (1999), no. 4, 1263–1290.

*Mathematical Institute, Utrecht University, P.O. Box 80010, 3508 TA Utrecht, the Netherlands, e-mail: sleijpen@math.uu.nl

†Delft Institute of Applied Mathematics, Delft University of Technology, Mekelweg 4, 2628 CD Delft, The Netherlands, e-mail: M.B.vanGijzen@tudelft.nl