Efficient Methods for Least-Norm Regularization

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The perturbed least-squares problem:

$$\min_x \|x\|, \quad s.t. \quad \|b - Ax\| \leq \epsilon$$

arises in the regularization of discrete forms of ill-posed problems when an estimate of the norm of the noise in the data is available. We derive sufficient optimality conditions for this problem and propose two different classes of algorithms: a factorization-based algorithm for small to medium-scale problems, and two matrix-free iterations for the large-scale case. Numerical results illustrating the performance of the methods demonstrate that both classes of algorithms are efficient, robust, and accurate. An interesting feature of our formulation is that there is no situation corresponding to the so-called hard case that occurs in the standard trust-region subproblem. Neither small singular values nor vanishing coefficients present any difficulty to solving the relevant secular equations. Moreover, both fixed and variable preconditioning can be incorporated in a straightforward manner.

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