IDR(s) for linear equations with multiple right-hand sides

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Krylov subspace methods such as the Bi-conjugate gradient (Bi-CG), Bi-CG STABilized (Bi-CGSTAB), Generalized Minimum RESidual (GMRES) and Quasi-Minimal Residual (QMR) methods are well-known and effective for solving linear systems $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ for $\boldsymbol{x}$, where $\mathbf{A}$ is a given $n \times n$ matrix, and $\boldsymbol{b}$ a given n-vector. We have many opportunities to solve linear equations with the same coefficient matrix and different right-hand sides (RHSs). Therefore, block Krylov subspace methods such as the block Bi-CG (Bl-BCG), block Bi-CGSTAB (Bl-BiCGSTAB), block GMRES (Bl-GMRES) and block QMR (Bl-QMR) methods have been developed for solving block linear systems $\mathbf{AX} = \mathbf{B}$ for $\mathbf{X}$, where $\mathbf{B}$ a given $n \times m$ matrix.

The Induced Dimension Reduction (IDR) method was originally developed in 1980 by Wesseling and Sonneveld. IDR(s) [3] has recently been proposed to improve the convergence of IDR, and it has been reported that IDR(s) is more effective than the hybrid BiCG methods. It is a natural idea to extend IDR(s) to block IDR(s) for solving the linear equation with multiple RHSs. Du et al. have proposed block IDR(s) [1] by using the same analogy as that of Bl-BiCGSTAB described by Sadok et al.

We propose IDR(s) for the linear equation with multiple RHSs by using an idea of seed systems. Our proposed IDR(s) is designed by extending the Bi-CGSTAB variant of IDR(s) described in [2]. Then in IDR step, bases of a Krylov subspace, which are orthogonal to an $n \times s$ initial shadow residual matrix, are generated by using an idea of seed systems. Our numerical experiments show that our proposed IDR(s) is effective.

References

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