Model Order Reduction for Dynamical Systems

Ph.D. Course

Instructor:	**Professor Danny C. Sorensen, Rice University, Houston, Texas, USA.**
Place:	**Collegezaal D, Mekelweg 4 (EWI building), Delft University of Technology.**
Date:	**November 1-5, 2010.**
Time:	**9am – 11:30am.**

Content: a subset of topics from the list below.

To help us plan, please register by sending an e-mail to Marielba Rojas (**marielba.rojas@tudelft.nl**).

There is no registration fee.

List of topics

I. Linear Systems Theory

1. A motivating example
2. Linear time invariant systems: exact solution
3. Laplace transform, the transfer function
4. Energy, controllability, observability
   - Input to state and state to output maps.
   - Energy: 0 to state (reachability) and Energy: state to 0
   - Controllability and Observability Definition
     (Plus Motivating Examples)
   - Distance to uncontrollability - Robust systems
   - Controllability and Observability Gramians
5. Norms
   - The $\mathcal{H}_2$ norm
   - The $\mathcal{H}_\infty$ norm
   - The Bode diagram
6. Spectral theory, stability, transient behavior
   - Spectrum, numerical range, pseudospectra
   - Asymptotic stability of linear systems
   - Transient growth of asymptotically stable linear, linearized systems

II. Moment Matching Algorithms

1. Rational approximation and moment matching
   - Model reduction as a rational approximation problem
   - The idea of moment matching
   - Pade approximation - optimality
2. Lanczos and Arnoldi processes
   - Krylov spaces, Controllability and Observability
   - Arnoldi, Symmetric Lanczos, Non-symmetric Lanczos
3. Matching moments for free
   - Fundamental Theorem of Krylov spaces
   - Moment Matching Arnoldi
   - Moment Matching NS-Lanczos
   - Cost and storage comparisons
   - Approximation properties
4. Rational Krylov process
   - Skelton's general interpolation result
   - Rational Krylov
   - Approximation properties
5. Moment matching with finite poles
6. Error bounds, stability considerations
   - The interpolation problem
   - A-posteriori bounds
   - Optimal interpolation - $\mathcal{H}_2$ bounds

III. Balanced Truncation Model Reduction

1. Gramian based model reduction and the SVD
   - Traditional Gramian : $\int \mathbf{x}\mathbf{x}^T \mathrm{d}t$
   - SVD truncation
   - Caveats on actual savings.
   - Specialized to LTI systems
2. Hankel singular values, balanced coordinates
   - The Hankel operator and Hankel Singular Values
   - Incorporating observations
3. Balanced truncation reduction
   - Moore's idea
   - The Lyapunov equation
   - The balancing transformation
   - Irrelevance of the state space coordinate system

4. Error bound, stability preservation
   - Lyapunov inertia theorem
   - Every truncation is controllable, observable, and asymptotically stable

IV. Numerical Solution of Lyapunov Equations

1. Existence, uniqueness, solution formulas
   - Spectrum of Lyapunov operator
   - Kronecker product linear system
   - Integral formulations of the solution
2. Direct solution methods
   - Bartels–Stewart, Hammarling, Sorensen–Zhou
3. Smith and ADI algorithms
   - Smith's method, Stein equation
   - Lyapunov/ADI connection
   - ADI iteration
   - Selection of ADI interpolation points
4. Decay of singular values of solutions
   - Smith/ADI gives singular value estimates
   - Rational approximation, condenser capacity, asymptotically-optimal shifts
   - Hermitian case, elliptic function bounds (briefly)
5. Low-rank Lyapunov solvers
   - Methods for truncating, updating SVDs (Baker)
   - Low-rank modified Smith method

V. Nonlinear Model Reduction

1. Proper Orthogonal Decomposition (POD)
   - Comparison to FEM
   - Examples and properties
2. Discrete Emperical Interpolation(DEIM)
   - Development of DEIM
   - Implementation Issues
   - Error Properties
   - Computational Examples